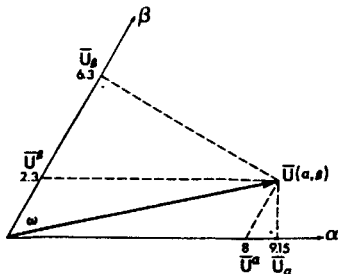
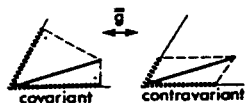


COVARIANT AND CONTRAVARIANT VECTOR COMPONENTS



GEOMETRY OF THE SPACE GIVES THEIR RELATION BY THE METRIC TENSOR



COVARIANT METRIC TENSOR

$$\bar{g}_{ij} = \sum_a \frac{\partial x^a}{\partial y^i} \frac{\partial x^a}{\partial y^j}$$

CONJUGATE TENSOR (CONTRAVARIANT METRIC)

$$\bar{g}^{ij} = \frac{\text{cofactor } \bar{g}_{ij}}{\text{determinant } g}$$

$$\bar{U}^i = \bar{g}^{ij} \bar{U}_j$$

$$\bar{U}_i = \bar{g}_{ij} \bar{U}^j$$

$$\begin{pmatrix} 2.3 \\ 8 \end{pmatrix} = \begin{pmatrix} 1.3 & -0.6 \\ -0.6 & 1.3 \end{pmatrix} \cdot \begin{pmatrix} 6.3 \\ 9.15 \end{pmatrix}$$

$$\begin{pmatrix} 6.3 \\ 9.15 \end{pmatrix} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2.3 \\ 8 \end{pmatrix}$$

Figure 1. The fundamental distinction in tensor network theory of the CNS of covariant and contravariant vectorial expressions and their relationship through the metric tensor. From Pellionisz and Llinás (1980).

senting points in Euclidean space. In theory of relativity, which concerns relativistic mechanics, the geometry of the vector-space is Riemannian. Tensor network theory of the central nervous system necessitates a further generalization of tensor theory beyond Euclidean and Riemannian spaces since functional multidimensional spaces of the CNS are not necessarily limited to such well-known geometries.

One of the most fundamental mathematical requirements that are necessary in tensor network theory of brain function is the explicit distinction of covariant and contravariant versions of a vector, since the intrinsic natural systems of coordinates of the CNS are composed of axes that are usually not orthogonal to one another, and these forms are identical only in orthogonal frames of reference. These different vectorial versions correspond to the independently established but non-executional covariant vector components, yielding a sensory intention-type vector, and to the physically executable but interdependent contravariant vector components, yielding a motor execution vector. The relationship between covariant and contravariant vectorial versions is characterized by the metric tensor (see Fig. 1).

The covariant intention to contravariant execution transformation (via the contravariant metric tensor or, in case of singularity, the Moore-Penrose generalized inverse of the covariant metric tensor) can be described in abstract tensorial notation, as well as by a particular network transformation in any given frame of reference. Such a quantitative neuronal network model is shown in Figure 2, transforming vestibular sensory

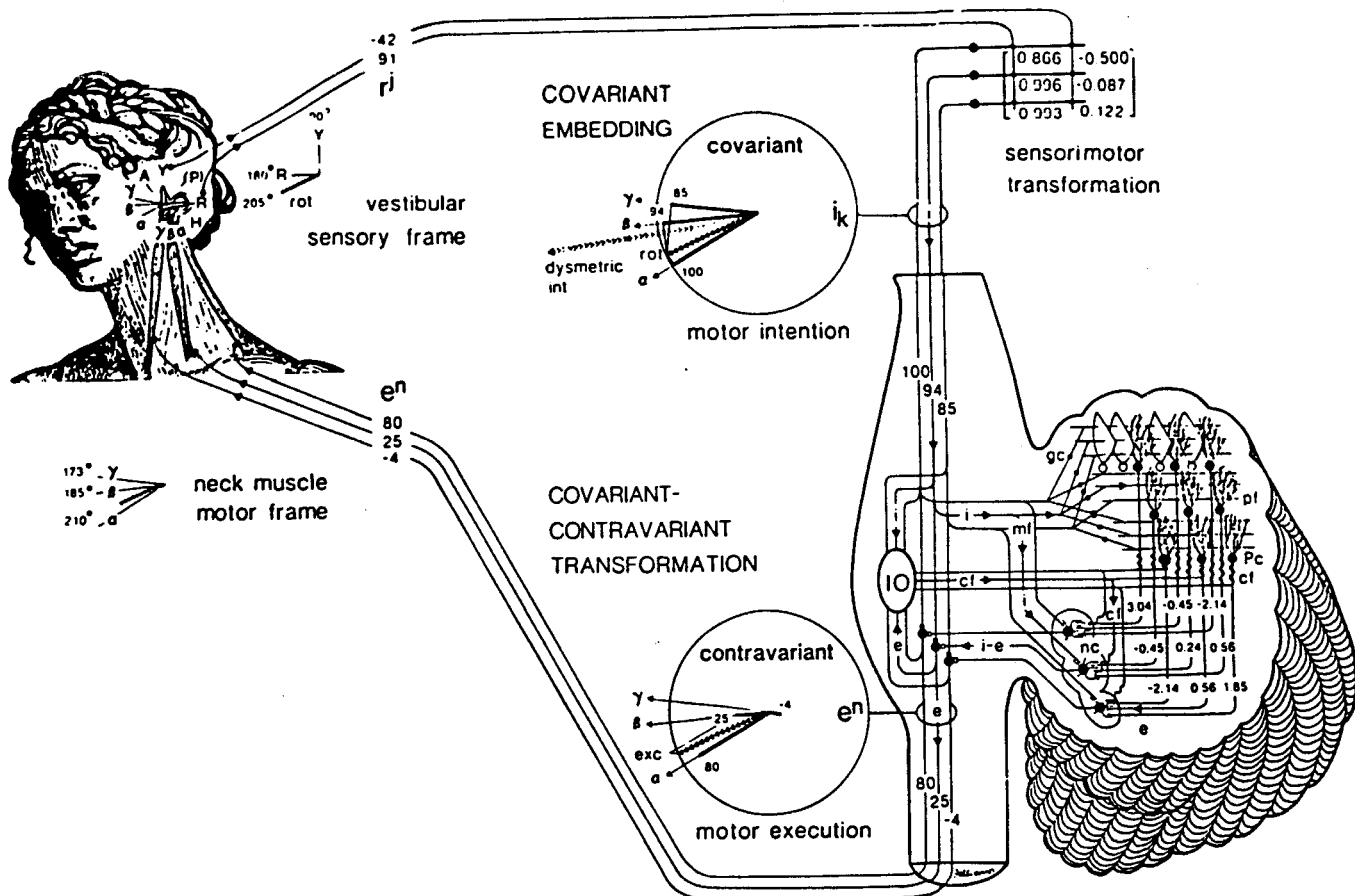


Figure 2. Tensor network model of the vestibulocollic reflex, embodying a covariant intention to contravariant motor execution transformation via the cerebellar neuronal network. From Pellionisz (1985).