



Figure 1 Covariant representation

so many units along another coordinate axis. Contravariance is very familiar to most scientists because it describes the way forces add in the familiar *parallelogram of forces*.

Notice that in the case of our familiar Euclidean coordinate system, the covariant and contravariant representations are the same, so this distinction is not needed.

With this distinction clear, Pellionisz and Llinás start to think about what it might mean. The contravariant description fits very well with the intuitive notion of the addition of forces produced by muscles, if each coordinate axis is identified with muscle motor activity. Because these individual forces add up like physical forces to produce a resultant, they are contravariant in nature.

The proprioceptive sensors act much more like the covariant vector representation. The covariant description would pick up the component of a force along the given coordinate axes. If we assume that the coordinate axes are identified with proprioceptors in a given muscle, then these sensors will respond only to force components along the muscle axis, that is, a muscle does not in general know what is going on in other muscles except through their components along the first muscle.

So the problem of sensorimotor transformation in these terms becomes one of relating a covariant sensory representation to a contravariant motor representation. But the whole system is connected together through the world — they are both looking at different aspects of the same thing: muscle forces and actions.

Pellionisz and Llinás suggest that the cerebellum might be the brain structure that closes the internal loop in the nervous system: That is, it transforms the sensory representation into the motor representation, mathematically, by transforming a covariant representation into a contravariant representation. Is there a standard mathematical way to describe this transformation that would give us insight into what the cerebellum might be doing?